Proc. SPIE. Vol.2363. 1995. P. 113-118

Generalized spectral-analytic method. Part II. Applications.

F.F.Dedus, S.A.Makhortykh, M.N.Ustinin

Institute of mathematical problems in biology,
Pushchino Moscow reg. 142292 RUSSIA

ABSTRACT

An application of the generalized spectral-analytic method, being developed by authors, in solving complex problems of signal processing and image analysis is described. Some particular problems are concereded. Some approaches to data compression, initial and boundary problems solution, dimention identification of planar curves are presented.

1.INTRODUCTION

Now we have efficient specialized methods for solving of various particular data analysis problems. In some cases application of these methods is justified by their fitting to concrete problems, which insures temporal and memory economy. However, when dealing with more complex problems, the efficiency of these techniques is lucking.

In real situations we have to solve not a single problem, but consequently a set of particular problems. For example, in the case of data arrays analysis the following steps are necessary: noise reduction, filtration, approximation, parameter estimation, simulation, organization of efficient data storing, *e.c.* Each step may require specific preprocessing and data representation. This factor leads to multiple data conversion and, may be, reserving of appropriate memory for data storing at each stage.

In these circumstances a unified computer technology is preferred, gain in computer time and memory economy.

The generalized spectral-analytic method is just developed for such problems. GSAM presupposes conversion from initial to unified representation in the choiced optimal function space. After this transforming, the above-mentioned tasks, are solved in parallel regime on the basis of the same data. At the same time, an essential reduction of data array size and, consequently memory volume used and computing time benefits are achieved.

Theoretical fundaments of GSAM are presented in [1]. The following part of the article is devoted to application of the approach to particular data analysis problems.

2.PROBLEMS OF DATA COMPRESSION AND FILTERING

When the initial signal is a set of N points f_i , we can obtain a compression effect by a simple procedure of the set decomposition in orthogonal series:

$$f(t) = \sum_{i=1}^{M} A_i T_i(t). \tag{1}$$

In this case the coefficient of compression is a simple ratio

$$K_c = \frac{M v_A}{N v_f}. \tag{2}$$

Here v_A and v_f are sizes of representation of values f_i and A_i . The upper limit in (1) is defined by the approximation accuracy.

This approach to the problem permits to eliminate both statistic and semantic redundancy. On its basis the adaptive numerical-analytic algorithms are built. The possibility to obtain an optimal description is connected with the large number of modified classic orthogonal bases.

An integration procedure for Fourier coefficients calculation leads to smoothing of high-frequency components of a signal. Thus, orthogonal quasi-reversible compression has a property to filter high-frequency noise and permits to construct noise-resistant algorithms. Calculation of the Fourier coefficients has one more advantage -- the stiffness of expansion and independence of coefficients from each other.

This method of information compression may be realized on analog analyzer or by direct computer calculations. In these cases both regular (non-adaptive) and adaptive algorithms may be constructed. Regular compression algorithms are applied, when the class of considered functions is known *a priori*.

The most efficient approach to the problem of information compression is based on digital spectral analyzer for simultaneous noise elimination and signal compression. Essentially, this is a problem of optimal filtering [2].

As an example we will bring the problem of geologic thomograms compression.

X-ray thomogram is a data array of 512×512 integer values (512K) of sample optic density. At the preprocessing stage initial information is brought to convenient representation for correct extraction of inclusion bounds (contours).

Initial file has complete information of the sample. However, in many applications only fragments of the image are interesting (*e.g.*, for which appropriate threshold conditions are fulfilled). Checking these conditions, extract these regions of the analyzed image.

Procedure of contour vectorization followed by expansion of its parametric representation in series leads to essential data amount compression. Depending on the task and the chosen image, compression coefficient (2) can be as great as 100.

Then, data in new representation are used for geometry, correlation, fractal analysis. Processing is carried on in space of expansion coefficients without signal restoration.

We must say, however, that, for example, the binary method of contour image compression gives a greater effect in comparison with spectral methods. But their specificity does not allow to use these methods effectively on other stages of image analysis. Here we may repeat a thesis of advantages of universal approaches for problem solving on a unified basis.

3.APPROXIMATION OF SOLUTIONS OF MATHEMATICAL PHYSICS PROBLEMS

Suppose, we have a problem

$$\begin{cases} \hat{L}y = f_1, \\ \hat{g}y = f_2. \end{cases} \tag{3}$$

Here $\hat{L}$ is a linear integral-differential operator (one-dimensional, for the sake of simplicity), $\hat{g}$ is the set of initial and/or boundary conditions, f_j, $j = 1, 2$ are known and y are unknown functions.

We will conceive that the functions y and f are expanded in one of the classic polynomials series.

$$y_N = \sum_{i=0}^{N} a_i T_i(t). \tag{4}$$

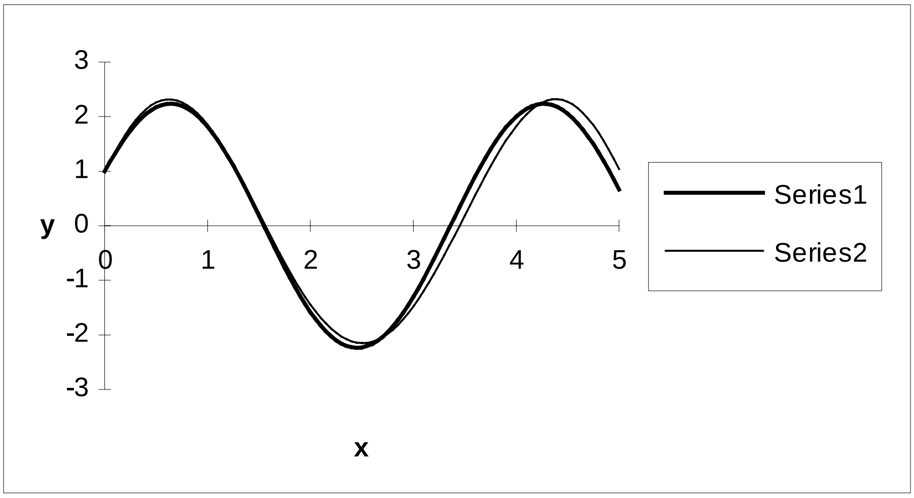
$$f_j(t) = \sum_{i=0}^{N} b_i^j T_i(t),$$

a_i are unknown coefficients, which must be found.

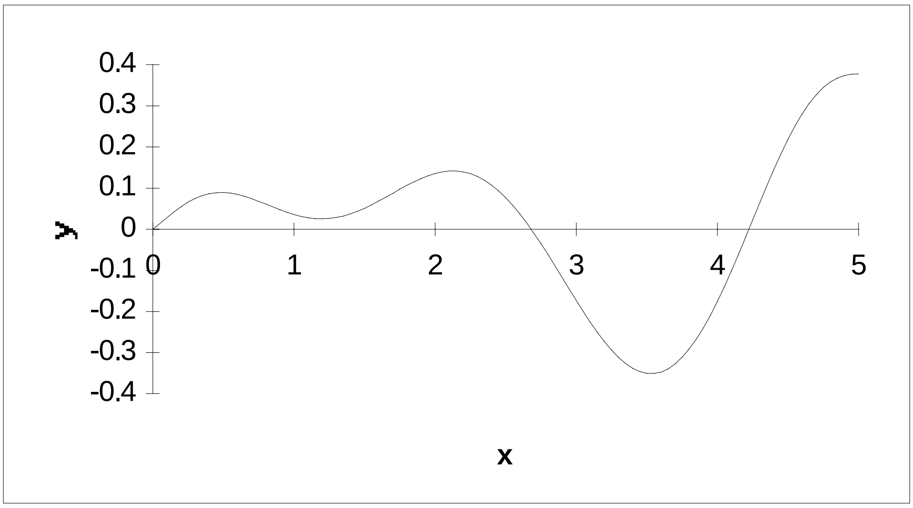
$$y'_N(t) = \sum_{i=0}^{N} d_i^{(1)} T_i(t),$$
$$\mathbf{d^{(1)}} = \mathbf{D_N a}, \tag{5}$$

here $\mathbf{d^{(1)}}$ and **a** are N and $N + 1$ - dimensional vector and $\mathbf{D_N}$ - $N + 1 \times N$ matrix.

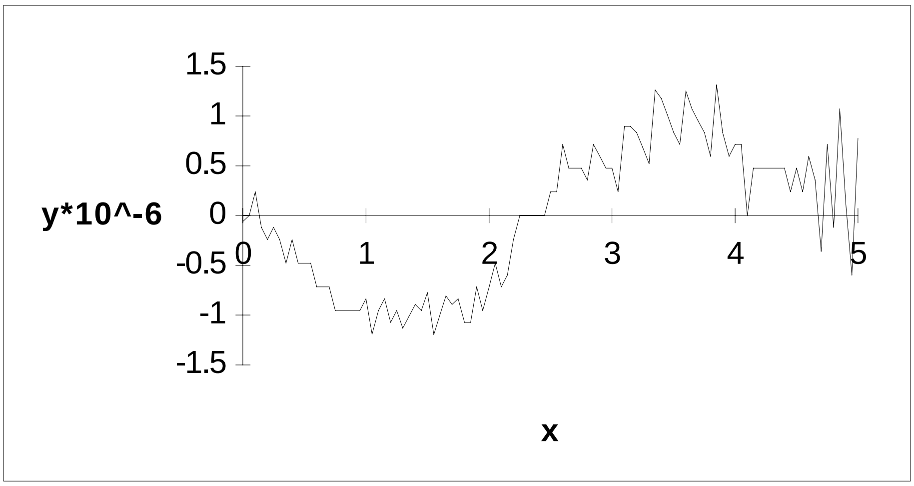
$$\mathbf{d^{(k)}} = \mathbf{D_{N-k+1}} \cdot \ldots \cdot \mathbf{D_N a}.$$



a)



b)



(c)

Figure 1. (a) Series 1 is exact solution of test problem and Series 2 is its approximation for *N=8*; error for *N=8* (b) and *N=16* (c).

Using the same method we find an approximation of an integration operator and, thus, an approximation of initial $\hat{L}$ (we should stress, that the operator may include expressions *s(t)y(t)*, where *s(t)* is a known function).

The described algorithm of construction of the operator approximation in orthogonal polynomials function subspace makes it possible to write an arithmetic system for unknown coefficients a_i. Missing equations are found by the same procedure from initial and boundary conditions. At this stage we have a closed problem for determination of needed coefficients a_i.

In the context of recognition and system classification, solution of the problem $\hat{L}y = f$ with supplementary conditions is equivalent to transformation of a recognition algorithm from *f*- to *y*-representation of the system.

Solution of the following test problem is presented in figure 1.

$$u'' + 3u = 0,$$
$$u(0) = 1,$$
$$u'(5) = 2\sqrt{3}\cos(5\sqrt{3}) - \sqrt{3}\sin(5\sqrt{3}).$$

The Chebyshev approximation is compared with the known analytic solution

$u(x) = \cos(\sqrt{3}x) + \sin(\sqrt{3}x).$

This approach was successfully used for the problem of Wiener-Hopf filtration and in 2D - case for the wave problem of inhomogeneous media acoustics.

<u>4.CALCULATION OF PLANAR CURVES DIMENTION</u>

Here some heuristic considerations on the application of GSAM to the problem of fractal dimension calculations will be presented.

In the case of sets on the plane this task appears in some problems of time series processing and image analysis.

Fractal time series with dimension D may be simulated on the interval $[0, +\infty)$ by Weierstrass-Mandelbrot function [3]

$$f(x) = \sum_{i=-n}^{n} \frac{\cos(b^i x)}{b^{2-D}} \qquad 1 < D < 2 \tag{6}$$

For finite interval [-1; +1] the following simulation is relevant

$$f(x) = \sum_{i=1}^{n} \frac{\cos(b^i \arccos(x))}{b^{2-D}}, \qquad 1 < D < 2 \tag{7}$$

In a special case of integer *b* we have Chebyshev expansion of *C(t)* (for integer *b* expression (6) corresponds to Fourier expansion).

If the upper limit in sum (7) is substituted by *N*, we have a prefractal of the order of *N* (some prefractals are presented in figure 2). It is characterized by the elements of width $\lambda \approx \frac{1}{b^N}$ and of height $h \approx \frac{1}{b^{N(2-D)}}$.

The length of element is $l \approx \frac{1}{b^N}\sqrt{1 + \frac{4b^{2N}}{b^{2N(2-D)}}}$, the total number of elements is $\nu \approx b^N$, then the total length of the fractal curve has the form

$$L = \nu l \approx \frac{2b^N}{b^{N(2-D)}} \propto \delta^{1-D}.$$

Here $\delta = b^{-N}$. According to the definition of fractal dimension, the value *D* is a dimension of the curve *f(t)*.

Let us assume, that the curve *f(t)* is to be analyzed in terms of fractal properties.
We suppose, that this curve is expanded in series

$$f(t) = \sum_{i=0}^{N} A_i T_i(t),$$

here $T_i(t) = \cos it$ or $\cos(i \cdot \arccos t)$. Let us assume, that the following condition is fulfilled: only coefficients A_{n_j} noticeably differ from zero and the approximation

$$n_j = b^j \qquad (8)$$

is relevant. If

$$\frac{A_{n_{j+1}}}{A_{n_j}} \propto b^{2-D}, \qquad (9)$$

then *D* is an estimation of curve fractal dimension.

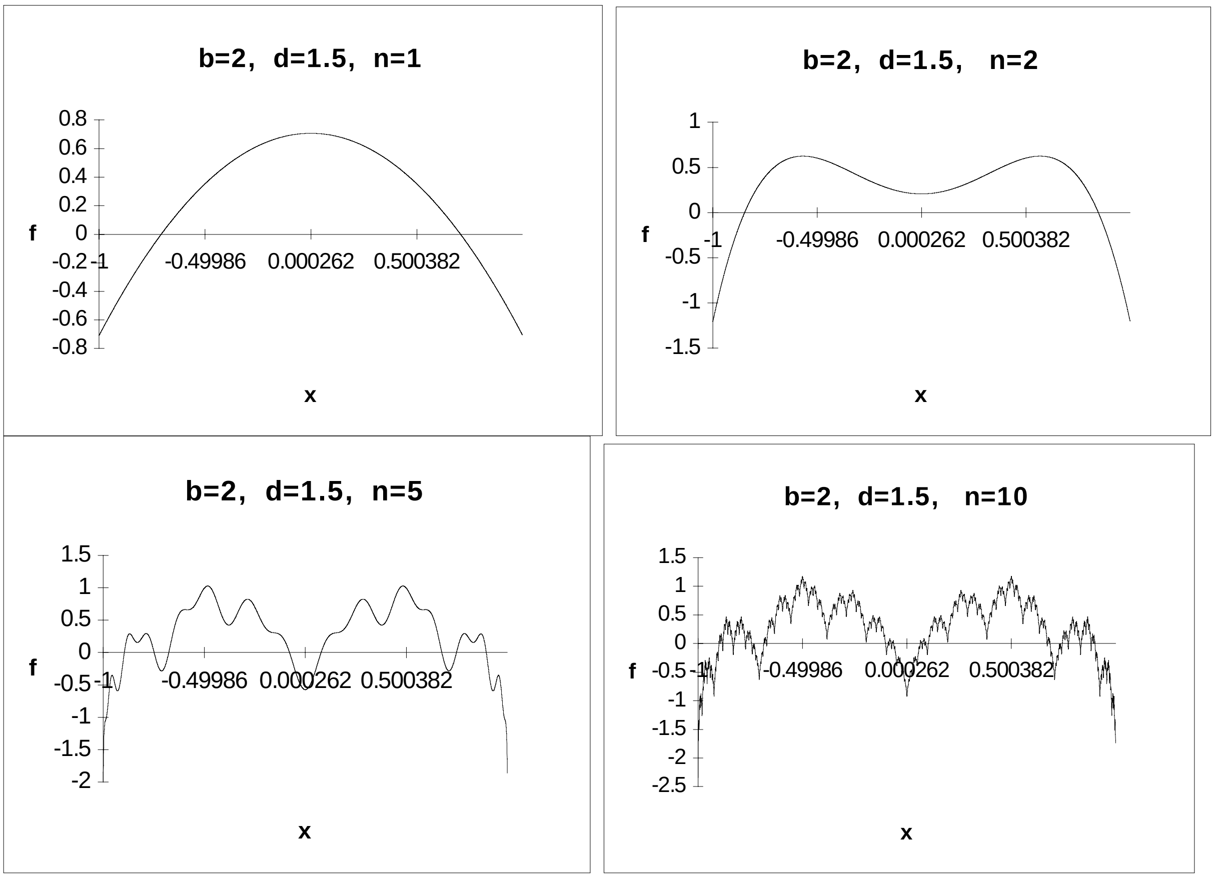


Fig. 2. Function $f(x) = \sum_{i=1}^{n} \frac{\cos(b^i \arccos(x))}{b^{2-d}}$.

In real practice expressions (8) and (9) are not exact but exist as persistence to appropriate exponential approximation. To use this technique to real curves, it must be calibrated on test data arrays. For example, this graduation may be fulfilled on the test signals in the form (6) for expansion (7) with various *D*. The results are presented on fig. 3.

Horizontal axis corresponds to actual dimension and vertical axis -- to its estimation. Using fig. 3, we can estimate dimension of each preassigned curve.

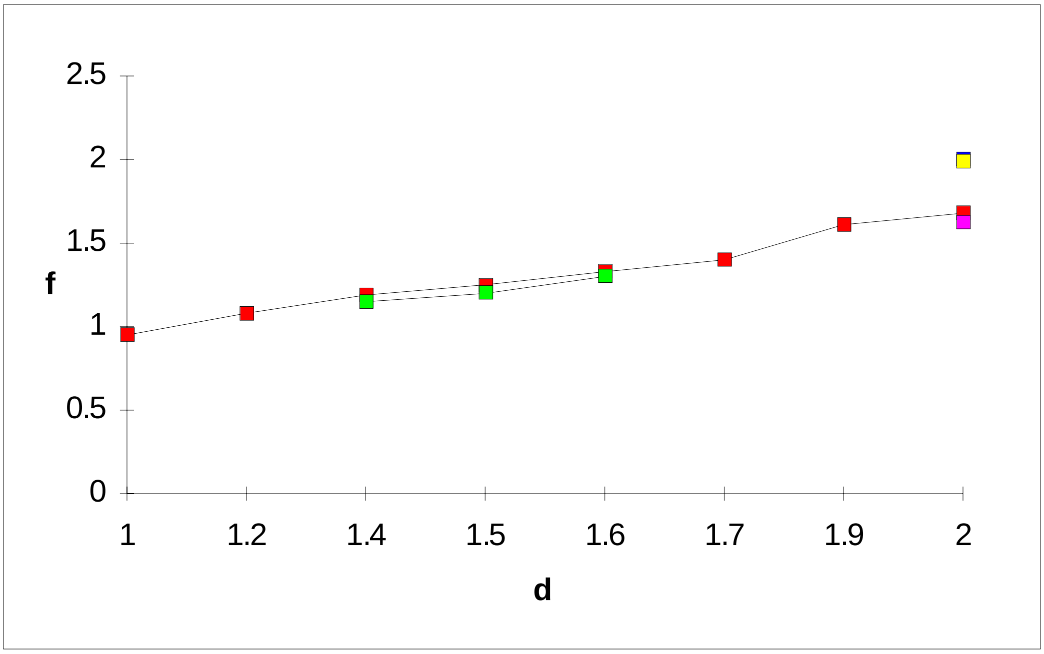


Fig. 3. Dependence of dimension estimation, calculated on the basis of expansion in Chebyshev series (*f*), of true curve dimension *d*.

5.CONCLUSIONS

There were many other problems, which were solved on the basis of GSAM. Among them we mention the problem of optimal Wiener - Hopf filter [2], correlation problems, pattern recognition [4] and electronic cartography. This method demonstrates a particular efficiency for the complex problems, which are decomposable into several independent parts. On GSAM basis multifunctional computerized system for signal processing, image analysis and pattern recognition is being built.

6.ACKNOWLEDGEMENT

This work is partly supported by Foundation of fundamental researches of Russian academy of sciences, grant ¹ 94-01-00226-a.

7.REFERENCES

1.A.F.Dedus, F.F.Dedus, S.A.Makhortykh, M.N.Ustinin, "Generalized spectral-analytic method. Part I. Theoretical foundations". *See this manual.*

2.I.V.Biktasheva, F.F.Dedus, S.A.Makhortykh, "Noise filtering in contour image analysis" (*In Russian).* In *Matematicheskie metody raspoznavaniya obrazov (MMRO - 6).* Moscow, 1993, p. 76.

3.J.Feder, *Fractals,* Plenum Press, New York and London, 1988.

4. A.F.Dedus, F.F.Dedus, S.A.Makhortykh and M.N.Ustinin, "Analytical description of multidimensional signals for solving problems of pattern recognition and image analysis", Pattern Recognition and Image Analysis, Vol.3, No.4, pp.459-469, 1993.