Proc. SPIE. Vol.2363. 1995. P. 109-112

Generalized spectral-analytic method. Part I. Theoretical foundations.

A.F.Dedus, F.F.Dedus, S.A.Makhortykh, M.N.Ustinin

Institute of Mathematical Problems of Biology RAS
Pushchino, Moscow Region, 142292, Russia

ABSTRACT

A new data processing technology is proposed, based on the adaptive analytical description of digital arrays by truncated orthogonal series. Further data processing is performed in the space of the expansion coefficients. The approach combines moderate computing times with the full use of analytical methods in problems of data compression, description and analysis.

I. INTRODUCTION

The wide application of digital methods in data processing is determined by the fast progress in the computer hardware. At the same time, the advancement of the experimental equipment results in the growth of the data arrays. Usually, the number of computational operations in data processing is a nonlinear function of the data amount and the growth of the information arrays often reduces to zero the gain, provided by the computer speed increasing.

This contradiction is intrinsic for the technical progress and makes of current content the creation of combined digital-analytical methods of data processing. The essence of such methods lies in the dividing of data processing into two stages. At the first stage one digitally converts the experimental data to their analytical description, getting rid of the excessively large arrays sizes. At the second stage all necessary calculations are performed analytically, the desired estimates being derived in general form because of the fixed structure of the analytical representation.

For some years we have been working on the development of the generalized spectral-analytic method (GSAM), based on the application of the classical orthogonal polynomials of continuous and discrete arguments [1].

Orthogonal expansions into a series of classical orthogonal polynomials, in comparison with other methods of analytical data description (such as description by the least-square method, or by polynomials of best approximation, or by spline functions, atomic functions, etc.) have a number of far-reaching advantages:

- any functional dependence, experimentally obtained as a continuous curve or a digital data file, may be analytically described by a segment of an orthogonal series with an arbitrary preset accuracy in the root-mean-square sense;
- expansion into orthogonal series is rigid, that is, calculation of new expansion coefficients does not effect the previous calculations;
- classical orthogonal bases of continuous and discrete arguments have parameters, whose variation generates orthogonal bases with new properties [2].

The consistent algorithmic and computational realization of the above advantages is a foundation for the development of a new technology, allowing to solve different problems of data processing from the unified position.

2. PRINCIPAL STATEMENTS

The generalized spectral-analytic method is based on the following statements:

1. Parametric description (vectorization). The object under study is described by a system of parametric equations of one argument:

$$\{x_i = X_i(t); \qquad i=1,...n \tag{2.1}$$

For example, a signal representing the variation of some quantity in time is described by a single equation (n=1) and a plane curve (in particular, the image contour) is unambiguously described by a system of two equations:

$$x = X(t), \quad y = Y(t). \tag{2.2}$$

2. Approximation. Parametric equations (projections) are approximated by truncated orthogonal series with a preset accuracy (in the uniform or root-mean-square sense):

$$X_i(t) = \sum_{k=0}^{N_i} C_{ik} P_k(t), \tag{2.3}$$

where $P_k(t)$ are known functions.

In order to attain the preset accuracy with the minimal length of series (2.3), we need to select an appropriate system $\{P_k\}$. Such series constitute an analytical description of the original object. The required accuracy of description and the number of coefficients vary according to the problem in hand.

3. Further processing hinges on the fact that the expansion coefficients carry all essential information about the object. This means that, having found the expansion coefficients, we can calculate all the intrinsic characteristics of the object or use the coefficients as features for the recognition problem. The dimension of the feature space can be further reduced through the use of the statistical properties of expansion coefficients for a given class of images.

The development of data processing technology based on these statements is supported by the following considerations:

- the parametric description of multidimensional objects (for example, plane and space curves) reduces the processing of multidimensional data to a set of one-dimensional problems;
- the adaptive analytical description of each projection by the most suitable orthogonal polynomials results in a drastic reduction of the data-representation volume (called data compression);
- further data processing uses analytical manipulations that can be performed in advance, thus eliminating completely the need for multiple processing of numerical arrays;
- the suggested procedures ensure full processing of data in the compressed form without restoring original objects.

3. VECTORIZATION AND ANALYTICAL DESCRIPTION

To illustrate the technology proposed we can apply it to the analysis of the plane curve [3]. The use of parametric representation of such an object requires the accomplishment of certain additional procedures, namely:

- outlining the contour image of the object under study;
- vectorizing the image and rendering its projections to a form convenient for analytical description;
- numbering the contour image and determining the initial point of numbering;
- specifying the parameters of the contour image in a convenient coordinate system.

Contour outlining and vectorization are independent problems and are presently described in detail. In some problems of data processing (for example, in computer cartography) images are represented as sequences of pairs (x_i, y_i), which constitutes the desired parametrization. Special programs are used for the vectorization of raster images.

For closed contours, it is difficult to provide a standard choice of the reference point on the contour from which a parameter is reckoned. This problem can be solved by optimizing a certain functional, depending on the reference point.

At the next step the numerical arrays corresponding to *X(t)* and *Y(t)* should be approximated by analytical expressions in the form (2.3). Eventually, any plane curve can be described analytically with a desired accuracy by a system of equations such as

$$X(t) = \sum_{k=0}^{N_x} A_k P_k(t), \quad Y(t) = \sum_{k=0}^{N_y} B_k P_k(t), \tag{3.1}$$

where the expansion coefficients are given by the integrals

$$A_k = \int_a^b X(t) P_k(t) p(t) dt, \quad B_k = \int_a^b Y(t) P_k(t) p(t) dt, \tag{3.2}$$

and *p(t)* is the weight function of the selected base $\{P_k\}$.

The calculation of integrals (3.2) is the most critical and laborious operation in the spectral analysis. It is here that the change to known functions of one variable is effected, allowing to take full advantage of analytical manipulations. Gauss quadrature formulas optimal for given types of polynomials are used for calculation of integrals (3.2).

For example, Gauss-Legendre quadrature formula is

$$A_k = \int_{-1}^{1} X(t) P_k(t) dt \approx \sum_{j=1}^{n} \lambda_j X(t_j) P_k(t_j), \tag{3.3}$$

where the values of $X(t_j)$ at the quadrature nodes are found by interpolation. This recipe tangibly reduces the amount of calculations (by the order of magnitude for a 256×256 raster) without losing essential details and permits to obtain rather long approximation series if necessary (N_x, N_y ~1000).

Turning to the use of expansion coefficients as features, we obtain a space of dimension $N_x + N_y$, instead of the initial dimension $M \times M$, where M is the raster size. Another advantage of this approach is the weak dependence of the number of calculations on the raster size in the image, which is due to the fact that the number of points in the contour increases linearly with M.

Equations (3.1) describe the shapes of the plane configurations to be studied. The expansion length varies according to the required accuracy of contour description. Thus, fewer terms of the orthogonal series are needed to get a general idea of the object's shape, whereas a greater depth of expansion is necessary to take into account minute details.

The orthogonal functions in equations (3.1) may not be from the same family, but, in a number of cases, it is useful that they should belong to the same base.

4. OPTIMIZATION OF THE ANALYTIC DESCRIPTION

It is very important to choose the most suitable orthogonal base for the approximation of curves. As a rule, it is necessary to strive for the optimal analytical approximation of each curve under analysis. There are two steps of the analytic description optimization.

1. The choice of an appropriate coordinate system.For example, a circle is described by trygonometric functions, which require N_x, N_y ~10-15 for approximation in the Cartesian coordinate system and in polar coordinates it is accurately described by $N_\rho = 0, N_\varphi = 1$.

2. The choice of the proper base and the base parameters. It is evident, that for good approximation the resemblance between the shape of the examined curve projection and the first orthogonal functions of the chosen basis is significant. Automation of the analytic representation of the initial data with the most suitable orthogonal base requires introducing of a procedure which would select for every signal such a base out of the set of bases entered into the computer.

In order to estimate the input signal, the concept of a "form coefficient" was introduced:

$$K_f = \frac{\int_a^b X(t)\eta(t)dt}{\int_a^b X(t)\xi(t)dt}, \tag{4.1}$$

where $X(t)$ is the projection under study, and $\eta(t)$, $\xi(t)$ are known functions. As follows from the definition, (4.1) is a ratio of the scalar products of the examined signal by the known functions.

Computing the integrals, we obtain the value characterizing the respective contributions of the functions $\eta(t)$ and $\xi(t)$ into the $X(t)$. If $\eta(t)$ and $\xi(t)$ are chosen so as to estimate the "opposite" properties of the signal (for example, $\eta(t)$ decay and $\xi(t)$ increase, or the presence of the maximum and minimum in $X(t)$, etc.) then the ratio (4.1) will emphasize these properties of the signal.

Let $\eta(t)=T-t$ and $\xi(t)=t$ on the interval [0,T], then we obtain

$$K_{f1} = \frac{\int_0^T X(t)(T-t)dt}{\int_0^T X(t)tdt}. \tag{4.2}$$

The integral in the numerator of Eq.(4.2) estimates the magnitude of the linearly decaying component in X(t), and the integral in the denominator of (4.2) assesses the increasing component. The ratio (4.2) lays emphasis on the characteristic property of $X(t)$ and gives the following results:

if $K>1$, then $X(t)$ decays on $[0,T]$;

if $K<1$, then $X(t)$ increases on $[0,T]$;

if K~1, then $X(t)$ is periodical or quasi-constant.

To organize the optimal approximation we must first estimate the properties of the orthogonal bases by the first few weighted polynomials assumed by the form coefficient. During the processing of $X(t)$, the computer estimates its form and performs the approximation with the basis, the form coefficient of which is nearest to the form coefficient found for the signal in question.

Proc. SPIE. Vol.2363. 1995. P. 109-112

5. CONCLUSION

This paper addresses the formulation of fundamental problems and approaches to solving them in order to describe multidimensional signals in the space of coefficients of expansion in terms of classical orthogonal polynomials.

For more details of the pattern recognition using the above formalism, see [3].

As for the use of the image contour, it is one of the principal basic structures of image analysis within the structural approach to recognition. However, the analysis of sets of expansion coefficients belongs, in essence, to the statistical trend in recognition. We consider our approach to be one way of integrating the methods of structural and statistical trends in problems of image processing.

Some other applications of the generalized spectral-analytic method are described in [4].

6.ACKNOWLEDGEMENT

This work is partly supported by Foundation of fundamental researches of Russian academy of sciences, grant ¹ 94-01-00226-a.

7. REFERENCES

1. A.Erdelyi et al., *Higher transcendental functions*, Vol.2, Ch.10, McGraw-Hill Book Co., N.Y., 1953.
2. F.F.Dedus, A.F.Dedus, and M.N.Ustinin, "A new data processing technology for pattern recognition and image analysis problems", *Pattern Recognition and Image Analysis*, Vol.2, No.2, pp.195-207, 1992.
3. A.F.Dedus, F.F.Dedus, S.A.Makhortykh and M.N.Ustinin, "Analytical description of multidimensional signals for solving problems of pattern recognition and image analysis", *Pattern Recognition and Image Analysis*, Vol.3, No.4, pp.459-469, 1993.
4. F.F.Dedus, S.A.Makhortykh and M.N.Ustinin, "Generalized spectral-analytical method. Part II. Applications", *this edition.*